Enhanced Linear Chip Level Equalizer for MIMO CDMA Downlinks in STTD Mode

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Abstract—In this study, an enhanced chip-level linear equalizer is proposed for multiple-input multiple-out (MIMO) multi-code code-division multiple access (CDMA) systems in space-time transmit diversity (STTD) mode. By retaining inter-antenna interference (IAI) within the equalized signal in an optimized manner, the signal-to-noise ratio (SNR) of the STTD combiner output is maximized. Substantial performance improvements are shown in the simulation results.

Index Terms—Chip-level equalizer, code-division multiple access (CDMA), multiple-input multiple-output (MIMO), space-time transmit diversity (STTD).

I. INTRODUCTION

CONVENTIONAL chip-level linear equalizers are designed for single-input single-output (SISO) code-division multiple access (CDMA) systems, and hence mainly focus on mitigating the inter-chip interference (ICI) to restore the code orthogonality and suppress the multiple access interference (MAI). In modern cellular CDMA systems incorporated with multiple-input multiple output (MIMO) techniques, the same spreading codes are reused across multiple transmit antennas due to the limited cardinality of the orthogonal codes [1]. This code reuse poses many challenges to the design of advanced chip-level equalizers coping with MAI and inter-antenna interference (IAI) simultaneously in the MIMO CDMA system. Specifically, the straightforward extension of the conventional equalizer by treating MAI and IAI with equal weights often shows unsatisfactory performance, since the residual IAI within the same code channel achieves spreading gains after dispreading and thus has more significant impacts than MAI. For spatial multiplexing systems in which independent data streams are transmitted from different antennas, the link-level performance can be boosted by using enhanced equalizers [2][3], where the imbalanced weights between MAI and IAI are considered and the signal-to-noise ratio (SNR) of the despreader output is maximized.

More careful treatments are needed to design equalizers for the MIMO CDMA systems in the space-time transmit diversity (STTD) mode, where space-time codes like the Alamouti code [4] is used to improve the transmission reliability. Unlike spatial multiplexing, the STTD scheme transmits coded data streams carrying the same information from different antennas, and thus introduces the man-made IAI, which later can be removed by the space-time decoder or diversity combiner. Different from the simple extension of the conventional chip-level equalizer for STTD [5][6][7], the proposed algorithm aims at removing the ICI while keeping the man-made IAI within the equalized signal in a desired manner. Therefore, improved performance can be achieved by maximizing the SNR of the STTD combiner output.

Throughout this paper, bold letters denote matrices and column vectors. \( I_N \) denotes an \( N \times N \) identity matrix. The superscripts \( (\cdot)^T \), \( (\cdot)^H \) and \( (\cdot)^\dagger \) denote transpose, complex conjugate and complex conjugate transpose. The symbol \( \otimes \) denotes Kronecker product.

The rest of paper is organized as follows: Section II describes the system model; Section III introduces the enhanced chip level equalizer for STTD; Section IV presents numerical results and Section V concludes.

II. SYSTEM MODEL

We consider the downlink of a MIMO-CDMA system with two transmit antennas and \( N_r \) receive antennas operating over fading multi-path channels. At the transmitter end, the data symbols are divided into two groups consisting of \( K \) substreams each, where \( K \) is the number of used orthogonal channelization codes. First, each substream in the group is spread with assigned channelization code of spreading gain \( SF \). Then, all substreams within the \( m \) th group, \( m = 1, 2 \), are combined and scrambled with a pseudo-random long scrambling code, and transmitted through the \( m \) th transmit antenna. We assume that each transmit antenna uses the same transmit power, which is equally allocated to the \( K \) code channels. The chip signal at the \( m \) th transmit antenna is given by

\[
z_m(i + j \cdot SF) = \frac{\sigma_s}{\sqrt{K}} \sum_{k=1}^{K} s_k(i)x_k^m(j), \quad m = 1, 2
\]  

(1)

where \( i \) is the chip index, \( j \) is the symbol index, \( k \) is the index of the composite spreading code, which is equivalent to the product of the channelization code and the long scrambling code. Thus, \( s_k(i) \) is the \( i \)th chip of the \( k \)th spreading code and \( x_k^m(j) \) is the \( j \)th symbol of the \( k \)th code channel transmitted over \( m \)th transmit antenna. We assume that \( E[|s_k|^2] = 1 \) and \( E[|x_k^m|^2] = 1 \) so that \( \sigma_s^2 \) is the average chip energy of each transmit antenna. For simplicity, our model does not include any downlink overhead channels (e.g., the pilot channel).

Having experienced multi-path fading, the received chip-level signal at the \( n \)th receive antenna is given by
for \( n = 1, \ldots, N_R \), where \( L \) is the number of the chip-spaced multi-paths, \( h_{mn}(l) \) is the \( l \) th path channel coefficient between the \( m \) th transmit antenna and the \( n \) th receive antenna, and \( v_n(l) \sim \mathcal{CN}(0, \sigma_v^2) \) is the complex additive white Gaussian noise (AWGN) at the \( n \)th receive antenna.

At a given time window where the channel can be considered stationary, a block of signal spanning \( N \) chip intervals is sampled. We interleave the received samples from multiple receive antennas into an \( N_R \times N \)-dimensional vector

\[
y_b = [y_1(bN + 1), \ldots, y_{N_R}(bN + 1), \ldots, y_1(bN + N), \ldots, y_{N_R}(bN + N)]^T
\]

where \( b \) is the block index, \( z_b \) is the \( 2N \)-dimensional transmitted chip signal vector defined as

\[
z_b = [z_1(bN + 1), z_2(bN + 1), \ldots, z_1(bN + N), z_2(bN + N)]^T,
\]

\( \mathbf{v}_b \) is the \( N_R \times N \)-dimensional background noise vector defined as

\[
\mathbf{v}_b = [v_1(bN + 1), \ldots, v_{N_R}(bN + 1), \ldots, v_1(bN + N), \ldots, v_{N_R}(bN + N)]^T,
\]

and \( \mathbf{H} \) is the \( N_R \times 2N \) block-circulant channel matrix defined as (6) at the bottom of the page, where

\[
\mathbf{h}(l) = \begin{bmatrix} h_{1,1}(l) & h_{2,1}(l) \\ \vdots & \vdots \\ h_{1,N_R}(l) & h_{2,N_R}(l) \end{bmatrix}, \quad l = 0, \ldots, L - 1
\]

represents the MIMO channel response at the \( l \)th path. \( \mathbf{H} \) is the \( N_R \times N \times 2N \) matrix modeling the inter-block interference (IBI) effect and is defined as

\[
\mathbf{H} = \begin{bmatrix} 0 & 0 & h(1) & \cdots & h(1) \\ 0 & 0 & \vdots & \vdots & \vdots \\ h(L - 1) & \vdots & 0 & 0 & h(L - 1) \\ 0 & 0 & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}
\]

Equation (3) can be approximated as IBI-free under certain circumstances, e.g., when a cyclic prefix (CP) or zero padding is used so that the last \( 2(L - 1) \) elements of \( \mathbf{z}_{b-1} - \mathbf{z}_b \) are zeros, or an overlap-save technique [8] is used so that the edge effects are eliminated.

In the following, by assuming one of above techniques is applied and omitting the block index, we have

\[
y = \mathbf{H} \mathbf{z} + \mathbf{v}.
\]

A. STTD Mode

Assuming the \( 4 \)th code channel is used in the STTD model, we have the \( j \)th Alamouti codeword

\[
[x_k^2(j), x_k^2(j + 1), x_k^2(2j + 1)]^T = [d_1(j), d_2(j), -d_2(j), d_1(j)]^T,
\]

where \( d_1(j) \) and \( d_2(j) \) are information symbols fed into the STTD encoder.

III. ENHANCED CHIP-LEVEL EQUALIZER FOR STTD

The proposed STTD equalizer estimates the IAI-contained chip signal \( \mathbf{Gz} \), where \( \mathbf{G} \) is a block-diagonal matrix defined as

\[
\mathbf{G} = \mathbf{I}_N \otimes \mathbf{P},
\]

where

\[
\mathbf{P} = \begin{bmatrix} 1 + \alpha & ge^{-i\theta} \\ ge^{i\theta} & 1 - \alpha \end{bmatrix}, \quad g \geq 0, \alpha^2 + g^2 < 1
\]

is a positive definite Hermitian matrix to be optimized. Note that any \( 2 \times 2 \) positive definite Hermitian matrix can be represented by (12) after being properly normalized to have a trace of 2. With an enhanced chip-level equalizer, the rotated chip signal is estimated as

\[
\mathbf{Gz} = \mathbf{Wz},
\]

where \( \mathbf{W} \) can be designed with various optimization criteria, e.g., zero-forcing (ZF) or MMSE. In a special case when \( \mathbf{P} = \mathbf{I}_2 \), we have the conventional chip-level equalizer which aims to maximize the chip-level SNR. With modified chip-level equalizer, we estimate the rotated chip signal \( \mathbf{Gz} \) instead of \( \mathbf{z} \) for two reasons: 1) The intentionally retained IAI characterized by the rotation matrix \( \mathbf{G} \) can be completely removed by STTD combiner as shown below; 2) The matrix \( \mathbf{G} \) can be optimized to maximize the symbol-level decision variable SNR instead of the chip SNR, and hence leads to better demodulation performance.

A. STTD Combining

After the chip-level equalization, we can obtain the symbol estimates by correlating the equalization output with the assigned spreading code. For an Alamouti codeword \( \mathbf{x} = \)
[d_1, d_2, -d_2, d_1]^T$, the despreading result will be

\[
\begin{bmatrix}
\hat{q}_1 \\
\hat{q}_2 \\
\hat{q}_3 \\
(1 - \alpha)d_1^* - (1 + \alpha)d_2^*
\end{bmatrix} = \begin{bmatrix}
(1 + \alpha)d_1 + ge^{j\theta}d_2 \\
ge^{-j\theta}d_1^* + (1 - \alpha)d_2 \\
ge^{j\theta}d_1^* - (1 + \alpha)d_2^* \\
(1 - \alpha)d_1^* - ge^{-j\theta}d_2^*
\end{bmatrix} + \mathbf{e}, \tag{14}
\]

where \(\mathbf{e}\) is the symbol detection error vector. The STTD combining rule is simply

\[
\hat{d}_1 = (\hat{q}_1 + \hat{q}_3)/2 \tag{15}
\]

and

\[
\hat{d}_2 = (\hat{q}_2 - \hat{q}_3)/2. \tag{16}
\]

During the diversity combining, the intentionally retained IAI characterized by the rotation matrix \(\mathbf{G}\) is removed completely.

**B. Rotation Matrix Optimization**

When a ZF equalizer is applied,

\[
\mathbf{W}_{ZF} = \mathbf{G}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \tag{17}
\]

is the pseudo-inverse matrix of \(\mathbf{H} \mathbf{G}^{-1}\). The autocorrelation matrix of the resulting noise vector becomes

\[
\mathbf{R}_{ZF} = \sigma_n^2 \mathbf{G}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{G}^H. \tag{18}
\]

Since \(\mathbf{H}\) is a block-circulant matrix, we have

\[
\mathbf{H} = (\mathbf{D}^H \otimes \mathbf{I}_{N_R}) \cdot \text{diag}(\mathbf{F}_n)_{n=1}^N \cdot (\mathbf{D} \otimes \mathbf{I}_2), \tag{19}
\]

where \(\mathbf{D}\) is the \(N \times N\) unitary DFT matrix, \(\mathbf{F}_n\) is the \(N_R \times 2\) matrix representing the channel frequency response at the \(n\)th tone with the \(ij\)-th entry

\[
\mathbf{F}_n[i,j] = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h_{j,i}(l) e^{-j\frac{2\pi}{N}nl}. \tag{20}
\]

By substituting (19) into (17) and (18), we have

\[
\mathbf{W}_{ZF} = (\mathbf{D}^H \otimes \mathbf{I}_2) \cdot \text{diag}(\mathbf{P}(\mathbf{F}_n^H \mathbf{F}_n)^{-1}\mathbf{F}_n^H)_{n=1}^N \cdot (\mathbf{D} \otimes \mathbf{I}_{N_R}) \tag{21}
\]

and

\[
\mathbf{R}_{ZF} = (\mathbf{D}^H \otimes \mathbf{I}_2) \cdot \text{diag}([\sigma_n^2 \mathbf{P}(\mathbf{F}_n^H \mathbf{F}_n + \rho \mathbf{I}_2)^{-1}\mathbf{P})_{n=1}^N \cdot (\mathbf{D} \otimes \mathbf{I}_2). \tag{22}
\]

Define \(\mathbf{\Omega}_{ZF} = \sum_{n=1}^N (\mathbf{F}_n^H \mathbf{F}_n)^{-1}\). Since the trace operation is similarity-invariant, we have

\[
\text{tr}(\mathbf{R}_{ZF}) = \text{tr}(\text{diag}([\sigma_n^2 \mathbf{P}(\mathbf{F}_n^H \mathbf{F}_n)^{-1}\mathbf{P})_{n=1}^N
\]

\[
= \sigma_n^2 \cdot \text{tr}(\mathbf{P} \mathbf{\Omega}_{ZF} \mathbf{P}). \tag{23}
\]

Then, the average symbol level SNR after despreading and STTD combining can be calculated by

\[
\text{SNR}_{sym} = \frac{SF \cdot \sigma_n^2 \cdot \text{tr}^2(\mathbf{P})}{K \cdot \text{tr}(\mathbf{R}_{ZF})} \tag{24}
\]

Fact 1. After ZF equalizing, despreading and STTD combining, the maximal SNR\(_{sym}\) is achieved when

\[
\mathbf{P} = Q \mathbf{\Omega}_{ZF}^{-1}, \quad Q > 0, \tag{25}
\]

where \(Q\) is the normalization factor. We have \(\text{tr}(\mathbf{P}) = 2\) when

\[
Q = \frac{\text{det}(2 \mathbf{\Omega}_{ZF})}{\text{tr}(\mathbf{\Omega}_{ZF})}. \tag{26}
\]

**Proof.** See Appendix A.

Comparing the conventional scheme with our enhanced scheme, we have the following fact:

Fact 2. With ZF equalization, the average symbol-level SNR obtained via the optimal rotation and STTD combining is always greater than or equal to that of the conventional scheme, and the equality holds if and only if \(\mathbf{\Omega}_{ZF}\) is a scaled identity matrix.

**Proof.** See Appendix B.

The results obtained above can be extended to the MMSE equalizer. By minimizing \(E[(\mathbf{Gz} - \mathbf{Wy})^H(\mathbf{Gz} - \mathbf{Wy})]\), we have

\[
\mathbf{W}_{\text{MMSE}} = \mathbf{G}(\mathbf{H}^H \mathbf{H} + \rho \mathbf{I}_{2N})^{-1} \mathbf{H}^H \tag{27}
\]

\[
= (\mathbf{D}^H \otimes \mathbf{I}_2) \cdot \text{diag}(\mathbf{P}(\mathbf{F}_n^H \mathbf{F}_n + \rho \mathbf{I}_2)^{-1}\mathbf{F}_n^H)_{n=1}^N \cdot (\mathbf{D} \otimes \mathbf{I}_{N_R})
\]

with corresponding MSE matrix

\[
\mathbf{R}_{\text{MMSE}} = \sigma_n^2 \mathbf{G}(\mathbf{H}^H \mathbf{H} + \rho \mathbf{I}_{2N})^{-1} \mathbf{G}^H \tag{28}
\]

\[
= (\mathbf{D}^H \otimes \mathbf{I}_2) \cdot \text{diag}([\sigma_n^2 \mathbf{P}(\mathbf{F}_n^H \mathbf{F}_n + \rho \mathbf{I}_2)^{-1}\mathbf{P})_{n=1}^N \cdot (\mathbf{D} \otimes \mathbf{I}_2)
\]

where \(\rho = \sigma_n^2 / \sigma_n^2\). If we assume that the resulting error signal \((\mathbf{Gz} - \mathbf{W}_{\text{MMSE}}\mathbf{y})\) can be regarded as Gaussian noise, we have the average symbol-level SNR of the STTD combiner output as

![Block diagram of the proposed implementation for enhanced STTD equalizer.](image)
\[
SNR_{\text{sym}} = \frac{SF \cdot \sigma_d^2 \cdot \text{tr}^2(P)}{K \sigma_d^2 \cdot \text{tr}(P \Omega_{\text{MSE}} P)}
\]

where \(\Omega_{\text{MSE}} = \sum_{n=1}^{N} (F_n F_n^H + \rho L_n)^{-1} \). Following the same derivation as for the ZF equalizer, we have the optimal rotation matrix

\[
P = Q \Omega_{\text{MSE}}^{-1}, \quad Q > 0.
\]

It shall be noted that the Gaussian assumption of error signals from MMSE equalizers is rather problematic under certain scenarios. Nevertheless, we can ensure that our enhanced MMSE scheme is at least asymptotically optimal since the MMSE solution converges to ZF when \(\rho\) is small enough. In fact, our numerical results presented below indicate noticeable gains with the enhanced MMSE equalizer in the medium-and-high SNR regime.

C. Remarks on Implementation

By examining (17) and (27), we can see that the enhanced equalizer for STTD is simply the product of the rotation matrix \(G\) and the conventional chip-level equalizer. The proposed implementation diagram for the STTD equalizer is shown in Fig. 1.

Note that the matrix rotation block and the despreader block are interchangeable. We prefer to do the rotation after despreading to reduce the implementation complexity.

IV. Simulation Results

In our simulation, we assume the number of receive antennas is \(N_R = 2\), the block size is \(N = 512\) chips, the chip rate is 3.84 Mc/s and the spreading factor is \(SF = 16\). We further assume \(K = 10\) code channels, which are 16-QAM modulated. To remove IBI and approximate the channel matrix as block-circulant, we use the overlap-save technique and throw away 128 boundary chips for each equalized block. For simplicity, the Rayleigh block-fading multi-path channel model is applied. This implies that the channel response varies independently by block and remains unchanged within one block. The power delay profile of the multi-path fading is adopted from the chip-spaced pedestrian-A and pedestrian-B models [9], as shown in Table I.

<table>
<thead>
<tr>
<th>Pedestrian-A</th>
<th>Pedestrian-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>110</td>
<td>-9.7</td>
</tr>
<tr>
<td>190</td>
<td>-19.2</td>
</tr>
<tr>
<td>410</td>
<td>-22.8</td>
</tr>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

The pedestrian-A model, with the delay spread of about 3 chip durations, represents the test scenario where the channel is lightly dispersive in time. In contrast, the pedestrian-B model with the delay spread of about 15 chip durations, represents more severely time-dispersive fading scenario.

V. Conclusion

We have proposed an enhanced chip-level linear equalizer...
for MIMO CDMA systems in the STTD mode. By suppressing MAI while retaining IAI within the equalized signal in an optimized manner, the SNR of the STTD combiner output is maximized. Simulation results have shown substantial performance improvements, especially for lightly time dispersive fading channels.

APPENDIX A
PROOF OF FACT 1
Let \( P \) be in the form of (12) so that we have fixed signal power after STTD combining. The optimization problem reduces to minimizing \( \text{tr}(P \Omega_{ZF} P) \).
Assume that the positive definite matrix
\[
\Omega_{ZF} = \begin{bmatrix}
a & ce^{j\omega} \\
-\frac{ce^{-j\omega}}{b}
\end{bmatrix},
\]
where \( a, b > 0 \), \( c \geq 0 \) and \( ab - c^2 > 0 \). Then,
\[
\text{tr}(P \Omega_{ZF} P) = a(1 + \alpha)^2 + b(1 - \alpha)^2 + (a + b)g^2 + 4cg \cos(\theta - \omega).
\]
(32)
To minimize \( \text{tr}(P \Omega_{ZF} P) \), we should have \( \theta = \pi + \omega \), i.e., \( \cos(\theta - \omega) = -1 \), and
\[
\text{tr}(P \Omega_{ZF} P) = a(1 + \alpha)^2 + b(1 - \alpha)^2 + (a + b)g^2 - 4cg.
\]
(33)
Take partial derivatives with respect to \( \alpha \) and \( g \) and set them to zero. By solving the resulting system of equations, we have
\[
\alpha = \frac{b+g}{a+b} \quad \text{and} \quad g = \frac{a+2b}{a+b}.
\]
Substituting the obtained \( \alpha \), \( \alpha \) and \( g \) into (12) leads to the optimal rotation matrix
\[
P = \frac{2}{a+b} \begin{bmatrix}
b & -ce^{j\omega} \\
-\frac{ce^{-j\omega}}{a}
\end{bmatrix} \frac{\Omega_{ZF}^{-1}}{\text{tr}(\Omega_{ZF})}.
\]
(34)
APPENDIX B
PROOF OF FACT 2
By substituting \( P = Q \Omega_{ZF}^{-1} \) and \( P = I_2 \) into (24), we obtain the symbol SNRs of the newly-proposed and conventional schemes, respectively. Denote the ratio between the two SNRs as \( \beta \). We have
\[
\beta = \frac{\text{tr}(\Omega_{ZF}) \text{tr}(\Omega_{ZF}^{-1})}{4}.
\]
(35)
Since \( \text{tr}(\Omega_{ZF}) = \lambda_1 + \lambda_2 \) and \( \text{tr}(\Omega_{ZF}^{-1}) = \lambda_1^{-1} + \lambda_2^{-1} \), where \( \lambda_1 \) and \( \lambda_2 \) are positive real eigenvalues of \( \Omega_{ZF} \), we can represent \( \beta \) as
\[
\beta = \left( \frac{\lambda_1 + \lambda_2}{\sqrt{\lambda_1 \lambda_2}} \right)^2,
\]
(36)
where \( \frac{\lambda_1 + \lambda_2}{2} \) and \( \sqrt{\lambda_1 \lambda_2} \) are the arithmetic and geometric means of \( \lambda_1 \) and \( \lambda_2 \), respectively. It is well-known that for a list of non-negative real numbers, its arithmetic mean is greater than or equal to its geometric mean; and further, that the two means are equal if and only if every number in the list is the same. Hence, we have
\[
\beta > 1 \quad \text{if} \quad \lambda_1 \neq \lambda_2,
\]
\[
\beta = 1 \quad \text{if} \quad \lambda_1 = \lambda_2.
\]
(37)
Note that \( \lambda_1 = \lambda_2 \) if and only if \( \Omega_{ZF} = \lambda_1 I_2 \).

REFERENCES