A Comparison of Normalizations for ZF precoded MU-MIMO systems in multipath fading channels

Monisha Ghosh, Senior Member, IEEE

Abstract—We compare two commonly used normalizations for zero-forcing (ZF) precoded multi-user multi-input-multi-output (MU-MIMO) systems: equal transmit power (ETP) and equal receive power (ERP) normalization. Both have been proposed in the literature, but with no comparison of the performance in multipath fading channels. We show analytically, and through simulations, that the ETP normalization leads to a higher total received power at the users and a higher sum rate. We also present simulation results in a multipath fading channel to corroborate the analysis and observe that the performance spread among users increases as the delay spread decreases with ETP normalization. This can be used to choose the appropriate normalization in a particular multipath scenario.

Index Terms—MU-MIMO, zero forcing, WLAN, precoding

I. INTRODUCTION

Multi-user multiple-input-multiple-output (MU-MIMO) techniques are considered essential for next generation wireless systems for increased throughput. While non-linear precoding methods, such as dirty paper coding [1] and Tomlinson-Harashima (TH) precoding [2] have been shown to have superior performance, their complexity is high. Hence, current standards, such as the draft IEEE 802.11ac specification [3], have only specified linear precoding methods, without mandating that any one particular precoding method be used. The system designer can use any linear precoding method with the power normalization chosen to satisfy desired metrics.

The most commonly used linear precoding method when each user has a single receive antenna is the zero-forcing (ZF) method which ensures that there is no multi-user interference at each user. With ZF precoding, there are optimal water-filling methods that can be used to maximize throughput. However these methods are usually complicated to implement and hence there are two power normalizations that are commonly used in practice: equal transmit power (ETP) and equal receive power (ERP) per user. Both have the same total transmit power over all users. In the literature, both methods are used. For example [4] uses ETP while [5] and [6] use ERP. However, in these and other papers, there is no explicit comparison made between these two methods, especially in multipath channels.

In this paper, we show analytically and through simulations with an IEEE 802.11ac physical layer (PHY) based on [3], that ETP normalization delivers a greater total power to users as compared to ERP as well as a greater sum rate. We corroborate this with packet-error-rate (PER) simulations and also show that when ETP normalization is used, the received signal to noise ratio (SNR) as well as the rate spread among the users depends on the root mean square (rms) delay spread of the channel. This observation can be used to choose the best normalization method for a given scenario.

The paper is organized as follows. Section II describes the MU-MIMO signal and channel model and derives the total received power and sum rates for both ETP and ERP normalizations for an OFDM system. Section III presents simulation results corroborating the analysis in Section II. Finally, conclusions are presented in Section IV.

II. MU-MIMO SIGNAL MODEL

A. MU-MIMO signal and channel model

In this paper we consider a linearly precoded MU-MIMO OFDM system with N sub-carriers, N_t transmit antennae, N_r users with a single antenna each and N_r ≤ N_t. The received signal vector \( \mathbf{r}_k \) across the N_r users on the k-th sub-carrier is:

\[
\mathbf{r}_k = \mathbf{H}_k \mathbf{Q}_k \mathbf{a}_k + \mathbf{n}_k, \quad k = 0, 1, \ldots, N-1
\]

where \( \mathbf{r}_k \) is a N_r×1 received vector, \( \mathbf{H}_k \) is the N_r×N_t channel matrix such that the \( i \)-th row represents the channel between the \( i \)-th user and the transmitter, \( \mathbf{Q}_k \) is the N_r×N_r precoding matrix, \( \mathbf{a}_k \) is the N_r×1 data vector where each element is assumed to be independent, identically distributed (i.i.d.) with unit variance, and \( \mathbf{n}_k \) is the N_r×1 complex, zero mean, additive white Gaussian noise (AWGN) vector where each element is assumed to be i.i.d. with variance \( \sigma_n^2 \).

When ZF precoding is used, the precoding matrix for the k-th sub-carrier can be derived as follows [4][5][6]. Let the pseudo inverse of the channel matrix be denoted as:

\[
\mathbf{Q}_k = \mathbf{H}_k^H \left( \mathbf{H}_k \mathbf{H}_k^H \right)^{-1}
\]

where the superscript “H” denotes conjugate-transpose. Let us now partition the matrix \( \mathbf{Q}_k \) into its N_r columns as follows:

\[
\mathbf{Q}_k = \begin{bmatrix}
\mathbf{q}_k^{(1)} & \mathbf{q}_k^{(2)} & \cdots & \mathbf{q}_k^{(N_r)}
\end{bmatrix}
\]

and define the magnitude squared of each column as follows:

\[
c_k^{(i)} = \mathbf{q}_k^{(i)} \mathbf{q}_k^{(i)^H}, \quad i = 1, 2, \ldots, N_r
\]

The ETP and ERP precoding matrices for each sub-carrier, k, are defined as follows:

\[
\mathbf{Q}_k^{(\text{ETP})} = \begin{bmatrix}
\mathbf{q}_k^{(1)} & \sqrt{\frac{1}{N_r}} \mathbf{q}_k^{(2)} & \cdots & \sqrt{\frac{1}{N_r}} \mathbf{q}_k^{(N_r)}
\end{bmatrix}
\]

and
Both precoding matrices $Q_k^{(ERP)}$ and $Q_k^{(ETP)}$ have the following properties: (i) multi-user interference at each user is zero since the $i$th row of the channel matrix $H_k$ is orthogonal to the $i$th column ($i \neq j$) of the precoding matrix and (ii) the total transmit power for both normalizations is the same and equal to the number of users $N_r$.

### B. Received power comparison

Since, by definition, the precoding matrices are orthogonal to the channel matrix ($H_k Q_k = I_{N_r}$), we have, for the $i$th user on the $k$th sub-carrier, the following expressions for the received signal:

$$r_k^{(i)} = \frac{1}{\sqrt{c_k^{(i)}}} a_k^{(i)} + \eta_k^{(i)}, \quad i = 1, 2, \ldots, N_r$$

and

$$r_k^{(i)} = \frac{N_r}{\sqrt{c_k^{(i)} + c_k^{(2)} + \cdots + c_k^{(N_r)}}} a_k^{(i)} + \eta_k^{(i)}, \quad i = 1, 2, \ldots, N_r$$

Hence, the average received power on the $k$th sub-carrier over all users for the ETP and ERP precoding matrices above are:

$$P_k^{(ETP)} = \frac{1}{N_r} \sum_{i=1}^{N_r} \frac{1}{c_k^{(i)}}$$

and

$$P_k^{(ERP)} = \frac{1}{N_r} \sum_{i=1}^{N_r} \frac{N_r}{c_k^{(i)} + c_k^{(2)} + \cdots + c_k^{(N_r)}} = \frac{N_r}{\sum_{i=1}^{N_r} c_k^{(i)}}$$

From the above equations, we see that $P_k^{(ETP)}$ is the reciprocal of the harmonic mean of the set of positive numbers $[c_k^{(1)}, \ldots, c_k^{(N_r)}]$ while $P_k^{(ERP)}$ is the reciprocal of the arithmetic mean of the same set of numbers. It is well known that the harmonic mean of a set of positive numbers is less than or equal to the arithmetic mean. Hence, we have:

$$P_k^{(ETP)} \geq P_k^{(ERP)}$$

The total received power, over all sub-carriers and all users is:

$$P^{(ETP)} \geq P^{(ERP)}$$

where $P^{(ETP)} = N_r \sum_{k=1}^{N} P_k^{(ETP)}$ and $P^{(ERP)} = N_r \sum_{k=1}^{N} P_k^{(ERP)}$.

The above result thus shows that, for the same total transmit power, the total received power across all users and all sub-carriers is greater with the ETP normalization rather than the ERP normalization when ZF precoding is used.

### C. Sum rate comparison

We can also compare the two normalizations on the basis of the sum rate. For the $k$th sub-carrier, the sum rate over all users for each of the normalizations can be derived from equations (7) and (8) as follows:

$$R_k^{(ETP)} = \sum_{i=1}^{N_r} \log_2 \left( 1 + \frac{1}{\sigma_k^{(i)}} c_k^{(i)} \right)$$

and

$$R_k^{(ERP)} = \sum_{i=1}^{N_r} \log_2 \left( 1 + \frac{N_r}{\sigma_k^{(i)}} \sum_{i=1}^{N_r} c_k^{(i)} \right)$$

Since $\log_2 (1 + \frac{1}{x})$ is a convex function of $x$ for $x > 0$, a straightforward application of Jensen’s inequality [7] gives:

$$\sum_{i=1}^{N_r} \log_2 \left( 1 + \frac{1}{\sigma_k^{(i)}} c_k^{(i)} \right) \geq N_r \log_2 \left( 1 + \frac{N_r}{\sigma_k^{(i)}} \right)$$

i.e.

$$R_k^{(ETP)} \geq R_k^{(ERP)}$$

Thus, the sum-rate across all users for each sub-carrier is greater when using the ETP normalization and hence the total sum rate (across all users and all sub-carriers) is also greater than that when using ERP normalization. i.e.

$$R^{(ETP)} \geq R^{(ERP)}$$

where $R^{(ETP)} = \sum_{k=1}^{N} R_k^{(ETP)}$ and $R^{(ERP)} = \sum_{k=1}^{N} R_k^{(ERP)}$.

### D. Multiple receive antennae

It has been shown in [8] that when each user has multiple receive antennae, the capacity maximizing precoder at high SNRs is derived from a block diagonalization method based on singular value decomposition (SVD). This process naturally leads to ETP for each user. Hence, the above comparison between ETP and ERP can be extended to multiple receive antennae and is omitted due to space constraints.

### III. SIMULATION RESULTS

In this section we present simulation results comparing the two normalizations. We assume a channel bandwidth of 40 MHz and a 128-FFT system (i.e. $N = 128$), 4 transmit antennae and 4 users with a single antenna each (i.e. $N_t = N_r = 4$). Perfect channel state information is assumed at both transmitter and receiver, where the transmitter knows the channels of all the single-antenna users and each user knows only its own channel. The ETP and ERP ZF precoder matrices are computed on each sub-carrier as described in equations (2) to (6). We assume the multipath channels from the transmitter antennae to the receiver antennae to be spatially uncorrelated with zero Doppler. In the time domain, each multipath channel is characterized by Rayleigh fading paths with an exponentially decaying power delay profile parameterized by the rms delay spread [9]. We compare performance for two values for rms...
delay spread, 15 ns and 50 ns. The total transmit power is normalized to be unity (0 dB) and the thermal noise variance $\sigma_n^2$ at each user is assumed to be the same.

A. Received power and sum rate comparisons

We generated 100000 channels according to the description above and computed the received instantaneous power with the two normalizations assuming a total transmit power of 0 dB. Figure 1 shows the cumulative distribution function (CDF) of the ratio $(P^{(ETP)}/P^{(ERP)})$ for 15 ns and 50 ns rms delay spread channels. Two conclusions can be drawn from these results: (i) as predicted from equation (12), $P^{(ETP)}$ is always greater than $P^{(ERP)}$ and (ii) the lower delay spread channel exhibits a wider variance in this difference than the higher delay spread channel. Figures 2 and 3 investigate this further by plotting the CDF of the ratio of the received power of the users to the total transmitted power (normalized to unity), sorted from best to worst according to their received instantaneous power for 15 ns and 50 ns rms delay spread respectively. Note, that the user ordering (best to worst) may be different for each channel realization. By definition, the ERP normalization will deliver the same received power to all users whereas with the ETP normalization each user receives a different power. It is interesting to note that the worst user with the ETP normalization receives almost the same power as the average user with ERP normalization for 50 ns rms delay spread and is only slightly worse for 15 ns rms delay spread. The difference between the best and worst user with ETP normalization is about 4.5 dB for 15 ns and 2.5 dB for 50 ns.

The sum rate difference $(R^{(ETP)} - R^{(ERP)})$ is shown in Figure 4. This difference is always positive, as predicted by equation (17) and the 15 ns rms delay spread channel has a larger variance. Figures 5 and 6 show the ergodic sum rate of the two normalizations for 15 ns and 50 ns rms delay spread respectively and we see again that the lower delay spread channel leads to a wider spread, about 2 bits/s/Hz among users when ETP normalization is used as compared to the 50 ns rms delay spread channel where the spread is about 1 bit/s/Hz.

B. Packet error rate (PER) comparisons

We simulated a MU-MIMO OFDM system based on the draft IEEE 802.11ac [3] PHY specification, with the same assumptions as above: a channel bandwidth of 40 MHz, 128-fft system, with bit-interleaved coded modulation (BICM) as specified in the standard. The metric of interest is packet error rate (PER), with 250 bytes per packet. The channel model is as described above and SNR is defined as $10\log_{10}(1/\sigma_n^2)$.

Figures 7 and 8 show the PER results with rate $\frac{3}{4}$ 16QAM for 15 ns and 50 ns rms delay spread respectively. We see that these results agree with the CDF results above, i.e. for lower delay spread channels (15 ns) there is a larger spread in performance among users (~5 dB) than with higher delay spread channels (50 ns) where the spread is about ~2 dB. Hence, when ETP normalization is used, the sum rate may be maximized by choosing a different modulation coding scheme (MCS) per user than the same one as shown here. Thus, these results indicate that link adaptation per user based on instantaneous received SNR and delay spread will maximize the achievable sum rate when using ETP normalization, while such per user link adaptation is not necessary for ERP normalization. Also, for larger delay spreads, ETP may not provide substantially larger sum-rate and if fairness is an important criterion for the application, ERP normalization may be preferred instead.

IV. CONCLUSIONS

In this paper we performed a comprehensive comparison of two commonly used normalizations, ETP and ERP, for ZF precoders. The main contributions are (i) we showed analytically that ETP delivers a higher sum-power and sum-
rate and (ii) we showed that in a realistic system the relative performance difference between ETP and ERP depends on the multipath delay spread. These results can be used by system designers to choose the best normalization given the application requirements and the multipath environment.

Figure 4 CDF of sum rate difference $(R^{\text{ETP}} - R^{\text{ERP}})$

Figure 5 Ergodic sum rate, 15 ns rms delay spread

Figure 6 Ergodic sum rate, 50 ns rms delay spread

REFERENCES


