Blind Estimation and Compensation of Frequency-Flat I/Q Imbalance Using Cyclostationarity

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Abstract— I/Q imbalance is one of the major concerns in the design of direct-conversion front-end receivers in high data rate wireless networks. To address the challenge, various I/Q imbalance estimation and compensation algorithms have been proposed in the literature. In this paper, we propose a blind cyclostationary method based estimation and compensation of frequency-flat I/Q imbalance. The proposed blind estimation algorithm uses second-order statistics to compensate I/Q imbalance, instead of estimating the mismatch parameters directly, and is an unbiased estimator when a DC offset exists at the receiver. The performance of our approach is evaluated and compared to other existing blind I/Q imbalance estimation algorithms.

Keywords: I/Q imbalance, Cyclostationary.

I. INTRODUCTION

The next generation wireless communication networks (for example, 3GPP LTE system) will provide high data rates such as 100 Mbps to subscribers. In commonly used direct-conversion front-end receiver, the received signal is I/Q downconverted from RF to baseband signal. Due to the imperfect oscillator in the RF front-end receiver, the I and Q signal paths will inevitably have different amplitudes and phases. This mismatch gives rise to image frequencies that become interference upon down conversion to baseband. The direct-conversion receiver is vulnerable to I/Q imbalance since I/Q separation is performed early in the RF/analog portion. We address the case of Direct-conversion Receiver (DCR) topology in this study, since DCR has benefits in terms of size and cost and is thus a preferred choice for higher levels of integration. The impact of I/Q imbalance is more severe to the system using high order modulations and high coding rates. Therefore, I/Q imbalance correction is essential for the design of higher data rates system [2].

In the past few years, extensive research has been done on I/Q imbalance estimation and compensation in wireless communication systems [4]-[7]. Methods such as hard decision (HD) approach [7] and statistical approach aim to estimate the mismatch parameters. The methods in [4]-[6] use second-order statistics of mismatched baseband equivalent to compensate I/Q imbalance, instead of estimating the mismatch parameters directly. In this paper, we improve the method in [4],[5] by using a cyclic auto-correlation based method that exploits cyclostationarity.

The rest of the paper is organized as follows. In Section II, the system model and I/Q imbalance model are described and formulated. In Section III, a one-tap compensator for frequency-flat I/Q imbalance is described. The cyclostationary property is reviewed in Section IV. A new I/Q imbalance estimator that uses auto-correlation method and exploits cyclostationary property is proposed in Section IV as well. In Section V, the numerical results are presented and discussed. Finally, conclusions are drawn in Section VI.

II. SYSTEM AND I/Q IMBALANCE MODEL

A. System Model

The received signal $z(t)$ is oversampled at a rate of $P/T$, where $T$ is the symbol duration and $P$ is an integer. Note that the sampling rate should be greater than the Nyquist rate, which implies that $P \geq 2$. We let $s[n]$ denote the transmitted modulated symbol, $g[n] = g(t)|_{t=nT/P}$ is the combined transmitter and receiver pulse shaping filter, $f_{\epsilon}$ denote the normalized carrier frequency offset with a uniform distribution between $[-\pi/2, \pi/2]$, and $\theta$ denotes the phase offset. We consider a multipath channel with $h[n,l]$ as the discrete channel impulse response and $L$ as the number of multipaths. Thus, we denote the following received discrete-time signal $z[n] = z(t)|_{t=nT/P}$ as

$$z[n] = e^{j(2\pi/P)f_{\epsilon}uT+\theta} \sum_{l=0}^{L-1} h[n,l]q[n-l]+v[n], \quad (1)$$

where complex additive noise $v(t)$ is assumed to be stationary but not necessarily white and/or Gaussian, and $q[n]$ is expressed as

$$q[n] = \sum_{u} s[n]g[n-uP]. \quad (2)$$

The analysis in the rest of the paper is based on the following assumptions [3]:
Assumption 1: $s[n]$ is a zero-mean independent identically distributed (i.i.d) sequence which is chosen from a finite-alphabet complex constellation with variance $\sigma_s^2$, i.e.,

$$E[s[m_1]s[m_2]] = \sigma_s^2 \delta[m_1 - m_2],$$

(3)

Assumption 2: The autocorrelation of a channel impulse response is given by

$$E[h[n_1, l_1]h^*[n_2, l_2]] = E[h[n_1, l_1]h^*[n_2, l_2]e^{j[l_1 - l_2]}].$$

$$= J_0(2\pi f_d T(n_1 - n_2)),$$

(4)

where $J_0(\cdot)$ is the zero-order Bessel function of first kind, and $f_d$ represents the maximum Doppler shift.

Assumption 3: $v[n]$ is a wide-sense stationary complex process independent of $h[n, l]$.

B. Frequency-flat I/Q Imbalance Model

The following assumptions are imposed on equation (3). In a quadratic (I/Q) direct-conversion receiver, the received signal $z(t)$ is translated to baseband by mixing it with a complex exponential generated by a local oscillator (LO) and a low pass filter as shown in Figure 1. In this architecture, the mixer, filters, amplifier and A/D converter are the source of I/Q component mismatch, due to their non-equal amplitude (gain) and phase imbalance (including imbalance at the transmitter side). In this paper, we assume the I/Q imbalance is frequency-flat over the entire receiver bandwidth, i.e., I/Q imbalance parameters (gain and phase imbalance) won’t vary in the entire receiver bandwidth. Hence, the corresponding mismatch received baseband signal $x[n]$ can be expressed as the received complex conjugate baseband signal $z[n]$ added with it’s own complex conjugate $z^*[n]$, i.e., $x[n]$ can be expressed as

$$x[n] = K_1(\Theta)z[n] + K_2(\Theta)z^*[n],$$

(5)

where $\Theta = [g, \phi]$ incorporates the amplitude mismatch $g$ and phase imbalance $\phi$, and $K_1(\Theta), K_2(\Theta)$ are complex numbers. To evaluate I/Q imbalance distortion to the received signal $z[n]$, we define the image-reject ratio (IRR) [4] as

$$\text{IRR} = \left|K_1(\Theta)\right|^2 / \left|K_2(\Theta)\right|^2.$$  

(6)

If there is no I/Q imbalance at the receiver, i.e., the term $K_2(\Theta)$ is equal to 0 then IRR $\rightarrow \infty$.

III. ONE-TAP COMPENSATOR FOR FREQUENCY-FLAT I/Q IMBALANCE

The I/Q imbalance compensator is trying to eliminate the conjugate signal (i.e., $z^*[n]$) from the received signal $x[n]$. Since frequency-flat I/Q imbalance is assumed in this paper, a simple one-tap compensator $w = -K_2^*(\Theta)/K_1(\Theta)$ can be applied to compensate the I/Q imbalance. For details of the derivation of the one-tap compensator, please refer to the Appendix. Let $\tilde{z}[n]$ denote the signal after I/Q compensation, which is given by

$$\tilde{z}[n] = x[n] + wz^*[n].$$

(7)

Applying the one-tap compensator $w = -K_2^*(\Theta)/K_1(\Theta)$ to the term $wz^*[n]$, we have

$$wz^*[n] = -K_2^*(\Theta)z^*[n] - \frac{K_2^2(\Theta)}{K_1(\Theta)}z[n].$$

(8)

Hence, plugging (8) into (7), we have the compensated signal $\tilde{z}[n]$ as

$$\tilde{z}[n] = (K_1(\Theta) - \frac{K_2^2(\Theta)}{K_1(\Theta)})z[n].$$

(9)

Therefore, compensation of frequency-flat I/Q imbalance does not require estimating I/Q imbalance parameters $\Theta$ (or $K_1(\Theta)$ and $K_2(\Theta)$). Instead, it only requires the estimation of $\Theta$. This motivates us to design a new estimation approach to solve the problem of I/Q imbalance estimation/compensation more robust with regard to any frequency-flat I/Q imbalance parameters $\Theta$.

IV. FREQUENCY-FLAT I/Q IMBALANCE ESTIMATOR USING CYCLOSTATIONARY APPROACH

A. Cyclic Correlation Property

The time-varying correlation of a general non-stationary process of $z[n]$ is defined as $R_{zz}(n; m) = E[z[n]z^*[n + m]]$, where $m$ is an integer lag. Signal $z(n)$ is second-order cyclostationary with period $P$ if only if there exists an integer $P$ such that $R_{zz}(n; m) = R_{zz}(n + kP; m)$, $\forall n, k$. To prove that $R_{zz}(n; m)$ is cyclostationary with period $P$, we examine if $R_{zz}(n; m) = R_{zz}(n + kP; m)$. Using (3), (4) and definition of $R_{zz}(n; m)$, we have

$$R_{zz}(n, m) = e^{-j(2\pi / \Theta)T} e^{j(l-l)m} \sum_{l} \frac{l-1}{l} J_0(2\pi f_d T) R_{qq}(n-l; m) + R_q(m).$$

(10)

It can be shown that the term $R_{qq}(n-l; m)$ is periodic with $P$. That is

$$R_{qq}(n-l+kP; m) = \sigma^2 \sum_u g[n-l+(k-u)P]g^*[n-l+m+(k-u)P] = \sigma^2 \sum_i g[n-l+iP]g^*[n-l-m+iP] R_{qq}(n-l; m).$$

(11)
Hence, we have \( R_{zz}(n;m) = R_{zz}(n + kP; m) \) for any \( n, k \). This means \( R_{zz}(n;m) \) is periodic with respect to \( n \) with period \( P \) for a fixed \( m \). Thus, \( R_{zz}(n;m) \) has discrete Fourier series coefficients given by
\[
F_{zz}(k;m) = \frac{1}{P} \sum_{n=0}^{P-1} R_{zz}(n;m)e^{-j(2\pi/P)kn}
\]
which are periodic with respect to \( k \) with period \( P \) [3]. \( F_{zz}(k;m) \) is called cyclic correlation and \( k \in \{-P/2, \ldots, P/2-1\} \) are called cyclic frequencies or cycles. From (12), \( F_{zz}(k;m) \) can be expressed as
\[
F_{zz}(k;m) = \frac{1}{P} \sum_{n=0}^{P-1} R_{zz}(n;m)e^{-j(2\pi/P)kn}
= e^{-j(2\pi/P)f_{mn}T_{mn}} \sum_{n} R_{qq}(n;m)e^{-j(2\pi/P)kn} + R_{vv}(m)\delta[k]. \quad (12)
\]

B. Proposed I/Q Imbalance Estimator

In the following, we propose a new approach that we estimate \( \frac{K_{2}^{*}(\Theta)}{K_{1}^{*}(\Theta)} \) by using cyclostationary approach. The proposed estimation algorithm is not limited to a particular air interface or waveform, and is therefore applicable to any wireless application. For simplicity of denotation, we drop the variable \( \Theta \) in \( K_{1}(\Theta) \) and \( K_{2}(\Theta) \), denoting them as \( K_{1} \) and \( K_{2} \) in the rest of the paper.

First, we evaluate the autocorrelation function \( R_{xx}(n;m) \) of \( x[n] \) and using the cyclic property of signal (i.e., \( E[x[n]z[n+m]] = 0 \)). We have
\[
R_{xx}(n;m) = E[x[n]x^{*}[n+m]]
= E[(K_{1}z[n] + K_{2}z^{*}[n])(K_{1}z^{*}[n+m] + K_{2}z[n+m])]
= \left| K_{1} \right|^{2} R_{zz}(n;m) + \left| K_{2} \right|^{2} \ast R_{zz}(n;m). \quad (13)
\]
Secondly, by evaluating conjugate autocorrelation function \( C_{xx}(m;m) \) of \( x[n] \), we have
\[
C_{xx}(m;m) = E[x[n]x^{*}[n+m]]
= E[(K_{1}z[n] + K_{2}z^{*}[n])(K_{1}z^{*}[n+m] + K_{2}z[n+m])]
= K_{1}K_{2}R_{zz}(n;m) + K_{1}K_{2}R_{zz}^{*}(n;m). \quad (14)
\]
In the special case where \( m = 0 \), the frequency offset \( f_{z} \) is decoupled from \( R_{zz}(n;0) \). Then, we have
\[
R_{zz}(n;0) = \gamma \sum_{l=0}^{L-1} J_{0}(0)R_{qq}(n-l;0) + \sigma_{v}^{2} = R_{zz}^{*}(n;0). \quad (15)
\]
Using (15), equations (13) and (14) are rewritten as
\[
R_{xx}(n;0) = \left( \left| K_{1} \right|^{2} + \left| K_{2} \right|^{2} \right) R_{zz}(n;0). \quad (16)
\]
\[
C_{xx}(n;0) = 2K_{1}K_{2}R_{zz}(n;0). \quad (17)
\]

Finally, since \( R_{zz}(n;m) \) is periodic with \( P \), \( R_{xx}(n;0) \) and \( C_{xx}(n;0) \) are periodic with \( P \) as well. The DFT of \( R_{xx}(n;0) \) and \( C_{xx}(n;0) \) are denoted as \( F_{xx}(k;0) \) and \( FC_{xx}(k;0) \) respectively, and are given by
\[
F_{xx}(k;0) = \sqrt{P} \left| K_{1} \right|^{2} + \left| K_{2} \right|^{2} \sum_{n=0}^{P-1} R_{xx}(n;0)e^{-j(2\pi/P)kn}. \quad (18)
\]
\[
FC_{xx}(k;0) = \frac{2}{P} K_{1} K_{2} \sum_{n=0}^{P-1} R_{xx}(n;0)e^{-j(2\pi/P)kn}. \quad (19)
\]
From (18), (19), we can choose cyclic frequencies \( k = 0 \) and \( -1 \) as parameters for the proposed one-tap I/Q imbalance compensator when root-rise cosine (RRC) pulse shaping filter is used [3]. The remaining cyclic frequencies are not chosen because they will degrade the estimation performance of \( R_{yy}(n;m) \) [3], [9].

Since \( \left| K_{1} \right|^{2} > \left| K_{2} \right|^{2} \), by using the (18) and (19), the estimate of \( K_{2}^{*} / K_{1}^{*} \) can be approximated as
\[
\frac{K_{2}^{*}}{K_{1}^{*}} = \left| \frac{K_{1}}{K_{2}} \right| = \frac{1}{2} \frac{FC_{xx}(k;0)}{F_{xx}(k;0)}. \quad (20)
\]
for \( k = -P/2, \ldots, 0, 1, \ldots, P/2-1 \).

Alternatively, using the method in [4], \( K_{2}^{*} / K_{1}^{*} \) can be expressed more precisely as the following
\[
\frac{K_{2}^{*}}{K_{1}^{*}} = \frac{FC_{xx}(k;0)}{F_{xx}(k;0) + \sqrt{\left| FC_{xx}(k;0) \right|^{2} - \left| C_{xx}(n;0) \right|^{2}}}. \quad (21)
\]

C. Impact of DC offset

If a DC offset in the receiver [8], then the right-hand side of (12) has to be increased by a term \( a^{2} \), where \( d \) is the DC offset amplitude. If cyclic frequency \( k = 0 \) is used in the proposed algorithm, the proposed algorithm in (21) becomes equivalent to Anttila’s algorithm as mentioned in (22). It was established in [5] that for Anttila’s algorithm there is a bias in I/Q imbalance estimation when a DC offset exists. Therefore, we choose not to use cyclic frequency \( k = 0 \) for the proposed algorithm in the presence of DC offset. For \( k = 1 \) and \( -1 \), the impact of DC offset will be removed because DC offset’s discrete Fourier series coefficient is a pulse only at \( k = 0 \) and is
zero elsewhere. Hence, the DC offset doesn’t impact the proposed algorithm in (20) or (21) with $k=1$ and -$1$.

V. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

The performance of the proposed I/Q imbalance compensation algorithm was simulated and compared to the moment-based blind estimation algorithm proposed by Anttila [4]. The simulation parameters are summarized in table 1. The simulation results are averaged by 1000 runs. Note that the performance of I/Q imbalance estimation depends not on the sampling rate but on the roll-off factor of the pulse shaping filter [10]. In the simulation, we adopt the frequency-flat I/Q imbalance model where $K_1(\Theta) = \frac{1+ge^{-j\phi}}{2}$, $K_2(\Theta) = \frac{1-ge^{j\phi}}{2}$, and $\Theta = [g, \phi]$.

<table>
<thead>
<tr>
<th>Transmission BW</th>
<th>QPSK, 16QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>QPSK, 16QAM</td>
</tr>
<tr>
<td>Mobility</td>
<td>3 km/hr</td>
</tr>
<tr>
<td>Channel model</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>Channel delay profile</td>
<td>[0, 0.5, 2.3] µs</td>
</tr>
<tr>
<td>Channel power profile</td>
<td>[-3, 0, -6] dB</td>
</tr>
<tr>
<td>Amplitude mismatch, $g$</td>
<td>1.07 dB</td>
</tr>
<tr>
<td>Phase imbalance, $\phi$</td>
<td>2 degree</td>
</tr>
<tr>
<td>Image-reject ratio (IRR) at analog front end</td>
<td>24 dB</td>
</tr>
<tr>
<td>DC offset amplitude</td>
<td>0 and 0.1</td>
</tr>
<tr>
<td>Pulse shaping filter</td>
<td>RRC with roll-off factor 0.22</td>
</tr>
<tr>
<td>Number of observed modulation symbols</td>
<td>10,000 symbols per simulation run</td>
</tr>
</tbody>
</table>

The IRR performance of I/Q imbalance estimation algorithms are plotted and compared in Figures 2-5 with different modulations, DC offset and SNR ranges. As shown in the QPSK modulation scenario in Figure 2, the proposed algorithm with cyclic frequency $k=1$ outperforms the Anttila’s algorithm at low SNR (4–9.4 dB) and underperforms at SNR above 9.4 dB. It is because Anttila’s estimate is unbiased at high SNR, however biased at low SNR. On the other hand, the proposed algorithm with cyclic frequency $k=0$ provides about the same IRR performance as Anttila’s algorithm in the entire range of SNR of interest. As shown in the 16QAM modulation scenario in Figure 3, the proposed algorithm with cyclic frequency $k=1$ outperforms the Anttila’s algorithm with SNR below 9.2 dB and underperforms with SNR above 9.2 dB. Similar to the trend observed in Figure 2, the proposed algorithm with cyclic frequency $k=0$ provides about the same performance as Anttila’s algorithm.

With the presence of DC-offset (as shown in Figures 4 and 5), the proposed algorithm with cyclic frequency $k=1$ yields better IRR performance than the Anttila’s algorithm at both low and high SNR regardless of modulation. This is because proposed algorithm is unbiased when DC-offset exists, while Anttila’s algorithm is biased. In the meanwhile, the proposed algorithm with cyclic frequency $k=1$ yields about the same IRR performance as the Anttila’s algorithm.

VI. CONCLUSIONS

In the paper, we proposed a new blind I/Q imbalance estimation algorithm that exploits cyclostationarity. The proposed blind estimation algorithm uses second-order statistics to compensate I/Q imbalance instead of estimating the mismatch parameters directly, and is an unbiased estimator when DC offset exists. The performance results show that the proposed algorithm is a very promising solution to I/Q imbalance estimation and compensation.

APPENDIX

The detailed mathematical derivation of the one-tap compensator in Section III is described here.

First, we have

$$\begin{align*}
\begin{bmatrix} x[n] \\ x'[n] \\ \end{bmatrix} &= \begin{bmatrix} K_1(\Theta) & K_2(\Theta) \end{bmatrix} \begin{bmatrix} z[n] \\ z^*[n] \end{bmatrix},
\end{align*}$$

(23)

Then, we can write (23) in the matrix form as $x = Kz$.

Due to the cyclostationary property of received signal $z$, we have $E[zr] = 0$. However, we have $E[xx'] \neq 0$ because of I/Q imbalance. Hence, we seek a compensation filter $w^T = [w_1, w_2]^T$ which restores the output $\hat{z} = w^T x$, which yields $E[\hat{z}r] = 0$. We have $E[\hat{z}z']$ expressed as

$$E[\hat{z}r] = w^T E[xr] = w^T KE[zz'],$$

(24)

where $E[zz']$ is given by

$$E[zz'] = E\left[\begin{bmatrix} z[n] \\ z'^*[n] \end{bmatrix} \begin{bmatrix} z[n] \\ z'^*[n] \end{bmatrix}^* \right] = \begin{bmatrix} E[z[n]z[n]] \\ E[z'^*[n]z[n]] \end{bmatrix} \begin{bmatrix} 0 \\ \sigma_z^2 \end{bmatrix},$$

(25)

Then, we have

$$E[\hat{z}z'] = w^T K \begin{bmatrix} 0 \\ \sigma_z^2 \end{bmatrix} = w_1K_2(\Theta) + w_2K_1^*(\Theta) = 0$$

(26)

Hence, we have the one-tap compensator given by

$$w = w_1 \frac{K_1(\Theta)}{K_2(\Theta)},$$

(27)

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Figure 1: Direct-conversion receiver structure.

Figure 2: IRR performance of the compensated signal with QPSK modulation at low SNR.

Figure 3: IRR performance of the compensated signal with 16 QAM modulation at high SNR.

Figure 4: IRR performance of the compensated signal with QPSK modulation at low SNR with DC offset.

Figure 5: IRR performance of the compensated signal with 16 QAM modulation at high SNR with DC offset.