Abstract—Energy detection is widely used in cognitive radios for spectrum sensing. Silent periods are required to make energy detection work properly. During a silent period, secondary users (SUs) remain silent so that the spectrum sensor does not confuse SU signals for primary user (PU) signals. Due to imperfect coordination, some SUs may transmit during a silent period, therefore causing possible false alarms at the energy detectors. In this paper, we propose to leverage matched filters that already exist in SUs to alleviate the impact of such SU interference. In particular, during a silent period, in addition to performing energy detection, an SU listens for any potential SU signal using an onboard matched filter already included in its communication hardware. As a result, the SU may gain the knowledge of whether other SUs have transmitted during a silent period. Such information can be combined with the sensing result from the energy detector for making a better decision on the detection of PU signals. For tractability, we focus on a simple cognitive radio system consisting of two SUs and two PUs. We propose an algorithm to combine the result from the matched filter and from the energy detector for the low SNR regime. The analysis shows that, for practical purposes, the proposed algorithm almost completely eliminates the negative impact of SU interference on the performance of energy detection.

I. INTRODUCTION

Spectrum sensing is an important function of a general cognitive radio [1](also called Secondary User or SU) since spectrum sensing allows a general cognitive radio to learn the radio environment for adaptation. For those cognitive radios operating in the TV broadcast channels, also known as TV Bands Devices (TVBDs), although the latest FCC ruling [2] does not require spectrum sensing, it does encourage the use of spectrum sensing because spectrum sensing has a number of advantages over the alternative approach, namely, the geo-location database only approach. In the database only approach, a cognitive radio needs to know its geo-location information before using the database. However, such information may not be always available (e.g., in certain indoor environments). Also, with the database only approach, a cognitive radio needs access to the Internet or other communication infrastructures, which may not be possible in some remote areas. In addition, the database only approach creates a single point of failure, leading to security concerns. As a result, it is possible that in the future, many cognitive radios will be equipped with a spectrum sensor.

Energy detection is widely used for spectrum sensing primarily because its use does not require a dedicated detector for each possible signal to be detected, in spite of its inferior performance compared to that of matched filtering among other shortcomings [3]. Since an energy detector does not differentiate the target signal from interfering signals, secondary users (SUs) must be silenced during the operation of an energy detector. The duration of the silent periods and how often they occur depend on the specific sensing algorithms and the target sensing performance. However, due to reasons such as loss of control messages and time synchronization errors, the coordination of silent periods may be imperfect. As a result, during a silent period, some SUs may transmit, causing a false alarm at the energy detector if no PU signals are present.

For cognitive radios capable of burst communication, such as IEEE 802.11 [4], matched filtering typically is used for detecting an incoming packet (also known as packet synchronization). That is, these cognitive radios already have a built-in matched filter which can detect the packets sent by other cognitive radios of the same radio access technology (RAT). By using the matched filter during a silent period, we obtain extra information on the channel activity almost for free. With such extra information, the spectrum sensing performance can be potentially improved.

In this paper, we propose an algorithm to leverage such matched filters to improve the performance of energy-detection based spectrum sensing. In particular, we focus on the low SNR regime, since that is a major challenge to spectrum sensing for cognitive radios. For a tractable analysis, we consider a simple cognitive radio system consisting of two SUs and two PUs. One of the SUs is a transmitter, and the other is a receiver with an energy-detection based spectrum sensor. One of the PUs is a transmitter, and the other is a passive receiver. Our analysis shows that the proposed algorithm can achieve almost ideal performance for scenarios of practical importance. In addition, the algorithm can be applied to a vast range of cognitive radios (with different RATs) and requires minimal hardware implementation effort.

The remainder of this paper is organized as follows. Section II describes the system model, Section III presents the proposed algorithm and the analysis, and Section IV concludes the paper.
II. SYSTEM MODEL

The system architecture is shown in Fig. 1. There are two Primary Users (PUs): a PU transmitter and a passive PU receiver. There are two Secondary Users (SUs): an SU transmitter (ST), and an SU receiver (SR) equipped with an energy-detection based Spectrum Sensor (SS) and a Sensing Processor (SP). The SU receiver uses matched filtering to detect incoming SU packets. The matched filtering result and the energy detection result are passed to the SP for making a final decision on the detection of a PU signal. The gain of the channel between the SU transmitter and the SU receiver is denoted by $c_s$, and between the PU transmitter and the SU receiver is denoted by $c_p$.

Due to imperfect coordination, the silent period used by the SU transmitter and the silent period used by the SU receiver may be out of phase, or even worse, the SU transmitter is unaware of a silent period. As a result, when the SU receiver is in a silent period and the SS performs energy detection, the SU transmitter may transmit, therefore causing a potential false alarm at the SS. Let the PU signal be $x_p(n)$ for $n = 1, ..., N$ with mean zero, and the SU signal be $x_s(n)$ with $x_s(n) = 0$ for $n = M + 1, ..., N$. This model captures the scenario where the SU signal may not span the entire observation window $N$ for energy detection. Among the $M$ samples of the SU signal, $L \gg 1$ are used for matched filtering. Such matched filtering could implement packet synchronization in practice for systems such as IEEE 802.11. The relationship between the PU signal duration, the SU signal duration $M$, the matched filter duration $L$.

III. ALGORITHMS

A. Energy Detection at the Spectrum Sensor

There are two possible cases: (1) in the ideal case, there is no SU signal, and this is the case usually considered in the literature; (2) in the practical case, SU signals may be present.

Ideal Case: The spectrum sensor may receive one of the following two possible signals:

\[ H_0: z(n) = w(n), \quad n = 1, ..., N \]  
\[ H_1: z(n) = x_p(n) + w(n), \quad n = 1, ..., N \]

where $N$ is the number of samples used, $w(n)$ are i.i.d Gaussian noise with mean zero and variance $\sigma^2_w$ and $x_p(n)$ are i.i.d. PU signal with mean zero and variance $\sigma^2_p$. The optimal energy detector is

\[ r := \sum_{n=1}^{N} |z(n)|^2 \geq \frac{\lambda}{H_0} \]

where $\lambda$ is the test threshold.

For large $N$, we have the probability of false alarm and the probability of detection [5]

\[ P_f^{H_0}(\lambda) = \int_{\lambda}^{\infty} P(r|H_0)dr = Q\left(\frac{\lambda - N\sigma^2_w}{\sqrt{2N}\sigma^2_w}\right) \]
\[ P_d^{H_1}(\lambda) = \int_{\lambda}^{\infty} P(r|H_1)dr = Q\left(\frac{\lambda - N(\sigma^2_s + \sigma^2_w)}{\sqrt{2N}(\sigma^2_s + \sigma^2_w)}\right) = Q\left(\frac{Q^{-1}(P_f^{H_0}) - \sqrt{N/2}\gamma_\cdot}{1 + \gamma_\cdot}\right), \]

respectively, where $\gamma_\cdot := \sigma^2_p/\sigma^2_w$ is the PU Signal-to-Noise Ratio (SNR) at the Spectrum Sensor (SS), and $Q(\cdot)$ is the Q function defined as $Q(t) = 1/\sqrt{2\pi} \int_{t}^{\infty} \exp(-x^2/2)dx$.

Non-ideal Case: In the non-ideal case, the SU transmitter may transmit during a silent period, and the spectrum sensor (SS) may actually receive one of the following four possible signals:

\[ h_0: z(n) = w(n) \]
\[ h_1: z(n) = x_p(n) + w(n) \]
\[ h_2: z(n) = x_s(n) + w(n) \]
\[ h_3: z(n) = x_p(n) + x_s(n) + w(n) \]

where $n = 1, ..., N$, $x_s(n)$ is the SU signal with mean zero and variance $\sigma^2_s$ for $1 \leq n \leq M$ and $x_s(n) = 0$ for $n > M$, $x_p(n)$ is the PU signal with mean zero and variance $\sigma^2_p$.
\( h_i \) are the hypotheses. As mentioned in Section II, this model captures the scenario where the duration of the SU signal could be less than the duration of the observation window for the energy detector. As an example of interpreting these equations, (5) says that, under \( h_0 \), i.e., if neither SU signal nor PU signal is transmitted, the SS will receive \( z(n) = w(n) \).

Since we have assumed that the SU mistakenly transmits during a silent period with probability \( q \) and such transmission is independent of the PU transmission, we have that
\[
P(h_0) = (1 - q) P(\text{PU does not transmit}) \quad \text{and} \quad P(h_2) = q P(\text{PU does not transmit}),
\]
where \( P \) denotes the probability. The SS is unaware of the existence of the SU signal and continues to use the decision regions used in the ideal case. A false alarm occurs if the SS decides \( H_1 \) when \( h_0 \) or \( h_2 \) has occurred, and by Lemma 1 (in the Appendix) we have the probability of false alarm
\[
P'_F(\lambda) = \frac{P(\text{SS decides } H_1| h_0 \cup h_2)}{P(h_0) + P(h_2)} P(\text{SS decides } H_1| h_0) + \frac{P(h_2)}{P(h_0) + P(h_2)} P(\text{SS decides } H_1| h_2) = (1 - q) P(\text{SS decides } H_1| h_0) + q P(\text{SS decides } H_1| h_2).
\]
\[
= (1 - q) Q\left( \frac{\lambda - N \sigma_w^2}{\sqrt{2N} \sigma_w^2} \right) + q Q\left( \frac{\lambda - M \sigma_s^2 - N \sigma_w^2}{\sqrt{2(M \sigma_s^2 + 2M \sigma_w^2 + N \sigma_s^2)}} \right),
\]
where the second equality follows from that \( h_0 \) and \( h_2 \) are mutually exclusive, and the probability of detection
\[
P'_D(\lambda) = \frac{P(\text{SS decides } H_1| h_1 \cup h_3)}{P(h_1) + P(h_3)} P(\text{SS decides } H_1| h_1) + \frac{P(h_3)}{P(h_1) + P(h_3)} P(\text{SS decides } H_1| h_3) = (1 - q) Q\left( \frac{\lambda - N \sigma_w^2 + \sigma_p^2}{\sqrt{2N} (\sigma_w^2 + \sigma_p^2)} \right) + q Q\left( \frac{(\lambda - M \sigma_s^2 - N (\sigma_w^2 + \sigma_p^2)) / \sqrt{2}}{\sqrt{M \sigma_s^2 + 2M \sigma_w^2 (\sigma_s^2 + \sigma_p^2) + N (\sigma_w^2 + \sigma_p^2)^2}} \right),
\]

B. Matched Filtering at the SU Receiver

There are two possible cases: (1) in the ideal case, there is no PU signal; (2) in the non-ideal case, PU signals may be present.

**Ideal Case:** The SU receiver (SR) receives
\[
A_0 : y(n) = v(n)
\]
\[
A_1 : y(n) = c_s x_s(n) + v(n)
\]
where \( n = 1, ..., L \leq M \), \( v(n) \) is i.i.d noise following a Gaussian distribution \( N(0, \sigma_n^2) \) and \( c_s > 0 \) is the channel gain. It is assumed that \( c_s x_s(n), n = 1, ..., L \) are known to the SR. The optimal detector is a matched filter [6]:
\[
T = \sum_{i=1}^{L} y(i) c_s x_s(i) \geq \eta, \quad H_0
\]
where \( \eta \) is the test threshold. Since \( c_s x_s(i) \) are known to the SR and \( y(i) \) is Gaussian under both hypotheses, \( T \) is also Gaussian. We have the probability of false alarm and the probability of detection [6]
\[
P'_F = \int_{-\infty}^{\infty} P(T|A_0) dT = Q\left( \frac{\eta}{\sqrt{L\sigma_c c_s \sigma_s}} \right)
\]
\[
P'_D = \int_{-\infty}^{\infty} P(T|A_1) dT = Q\left( \frac{\eta - L c_s^2 \sigma_s^2}{\sqrt{L\sigma_c c_s \sigma_s}} \right)
\]
\[
= Q\left( Q^{-1}(P'_F) - \sqrt{\eta L} \right),
\]
respectively, where \( \gamma_s := c_s^2 \sigma_s^2 / \sigma_v^2 \) is the SU SNR at the SR. 

**Non-ideal Case:** The SR receives one of the following four possible signals:
\[
h_0 : y(n) = v(n)
\]
\[
h_1 : y(n) = c_p x_p(n) + v(n)
\]
\[
h_2 : y(n) = c_s x_s(n) + v(n)
\]
\[
h_3 : y(n) = c_p x_p(n) + c_s x_s(n) + v(n)
\]
where \( n = 1, ..., L \), \( v(n) \) is the noise, \( x_p(n) \) is the PU signal, \( x_s(n) \) is the SU signal, and \( c_p > 0 \) satisfying \( c_p^2 \sigma_p^2 / \sigma_v^2 \ll 1 \) is the channel gain. Note that \( h_i \) in (16)-(19) and \( h_i \) in (5)-(8) refer to the same event. By applying Lemma 2 in the Appendix, we have for large \( L \)
\[
P'_F = P(\text{SR decides } A_1| h_0 \cup h_1) = (1 - q) Q\left( \frac{\eta}{\sqrt{L\sigma_v c_s \sigma_s}} \right) + q Q\left( \frac{\eta}{\sqrt{L\sigma_v c_s \sigma_s}} \right)
\]
\[
P'_D = P(\text{SR decides } A_1| h_2 \cup h_3) = (1 - q) Q\left( \frac{\eta - L c_s^2 \sigma_s^2}{\sqrt{L\sigma_v c_s \sigma_s}} \right) + q Q\left( \frac{\eta - L c_s^2 \sigma_s^2}{\sqrt{L\sigma_v c_s \sigma_s}} \right)
\]
where \( q \) is the priori probability that the PU uses the spectrum.

C. Proposed Algorithm for the Sensing Processor

During a silent period, the Spectrum Sensor (SS) performs energy detection. If the SU transmitter mistakenly transmits during the silent period, the SS will likely detect relatively significant energy because of the weak PU signal assumption and generate a false alarm. Such false alarm can be eliminated if the SU transmission can be identified. We propose to leverage the SU receiver to identify the SU transmission. Specifically, the SU receiver listens for SU signals via a matched filter during a silent period. This is possible because in many practical communication systems, a known synchronization sequence is sent before the transmission of the data portion of a packet, and a receiver detects a packet by applying a matched filter matched to the synchronization sequence.
If the SU receiver detects an SU signal, it reports the detection to the Sensing Processor (SP). The optimal strategy for the SP to take is to declare the absence of PU signals. To see this, let the probability that the SP decides $H_0$ be $p_0$. Recall the assumption of under utilization of the spectrum by the PU, i.e., $P(H_0) > P(H_1)$. The average probability of a decision error is $(1 - p_0)P(H_0) + p_0P(H_1)$ which is minimized with $p_0 = 1$. With this strategy, the probability of false alarm is

$$P_F(\zeta) = P(\text{SP decides } H_1| h_0 \cup h_2)$$

$$= (1-q)P(\text{SS decides } H_1|h_0)P(\text{SR decides } A_0|h_0)$$

$$+ qP(\text{SS decides } H_1|h_2)P(\text{SR decides } A_0|h_2)$$

$$= (1-q)Q \left( \frac{\zeta - N(q^2_p + \sigma_p^2)}{\sqrt{2N\sigma_w^2}} \right) \left( 1 - Q \left( \frac{\eta}{\sqrt{L(\sigma^2_w + c_q^2\sigma_s^2)}} \right) \right)$$

$$+ qQ \left( \frac{\zeta - M\sigma_s^2 - N\sigma_w^2}{\sqrt{2(M\sigma_s^2 + 2M\sigma_s^2\sigma_w^2 + N\sigma_w^2)}} \right) \left( 1 - Q \left( \frac{\eta - Lc_q^2\sigma_s^2}{\sqrt{L(\sigma^2_w + c_q^2\sigma_s^2)}} \right) \right),$$

where $\zeta$ is the threshold used by the SS, and the probability of detection is

$$P_D(\zeta) = P(\text{SP decides } H_1| h_1 \cup h_3)$$

$$= (1-q)P(\text{SS decides } H_1|h_1)P(\text{SR decides } A_0|h_1)$$

$$+ qP(\text{SS decides } H_1|h_3)P(\text{SR decides } A_0|h_3)$$

$$= (1-q)Q \left( \frac{\zeta - N(q^2_p + \sigma_p^2)}{\sqrt{2N\sigma_w^2}} \right) \left( 1 - Q \left( \frac{\eta}{\sqrt{L(\sigma^2_w + c_q^2\sigma_s^2)}} \right) \right)$$

$$+ qQ \left( \frac{(\zeta - M\sigma_s^2 - N\sigma_w^2)/\sqrt{2}}{\sqrt{M\sigma_s^2 + 2M\sigma_s^2(\sigma_w^2 + \sigma_w^2) + N(\sigma_w^2 + \sigma_w^2)}} \right) \left( 1 - Q \left( \frac{\eta - Lc_q^2\sigma_s^2}{\sqrt{L(\sigma^2_w + c_q^2\sigma_s^2)}} \right) \right).$$

We now analyze the performance of the proposed algorithm in connection with the other algorithms described above. We focus on the low SNR regime for the PU signal, i.e., $\gamma_p \ll 1$, since that is the most challenging scenario for spectrum sensing.

We first analyze the Receiver Operating Characteristic (ROC) [6] for the energy detector in the non-ideal case, i.e., the $P_D' - P_F'$ curve. We plot the curve in Fig. 3 for the following setting: $\gamma_p = -10\text{dB}$, $\gamma_s = 10\text{dB}$, $N = 3000$, $M = 1000$, $L = 100$, $c_p = 1$, and $q = 0.05$. The threshold $\eta$ for the matched filter in (14) is set such that $P_F' = 0.01$. It is easily seen from (10) and (9) that as $\lambda \to \infty$, $P'_D \to 0$ and $P_F' \to 0$ and hence the point $(0,0)$. Also, for $\lambda = 0$, $P'_F \to 1$ and $P'_D \to 1$ as $N \to \infty$ and hence the point $(1,1)$. However, we notice that there is an upward line segment starting from point $(q,q)$ with $P'_F$ tightly confined around $q$ for a wide range of values for $P_D'$. Why does this happen? We explain this in the following Theorem.

**Theorem 3.1:** For any given $P_D' \in (q,1)$, $\epsilon > 0$, there exist $N$ and $\lambda = (1 + \epsilon_1)N(\sigma^2_w + \sigma^2_p)$ where $\epsilon_1 \in (-\sigma^2_p/(\sigma^2_w + \sigma^2_p))$, such that $|P_F' - q| < \epsilon$.

**Proof:** We have $\lambda - N\sigma^2_s = N(\epsilon_1\sigma^2_w + (1 + \epsilon_1)\sigma^2_p)$. Since $\epsilon_1 > -\sigma^2_p/(\sigma^2_w + \sigma^2_p)$, we have that $(\lambda - N\sigma^2_s)/\sqrt{2N\sigma^2_s} \to \infty$ and hence $Q((\lambda - N\sigma^2_s)/\sqrt{2N\sigma^2_s}) \to 0$ as $N \to \infty$. Similarly, since $\epsilon_1 < \delta$ and $M\sigma^2_s > (1 + \delta)N(\sigma^2_w + \sigma^2_p)$, we have that

$$\lambda - M\sigma^2_s - N\sigma^2_w \to -\infty,$$  as $N \to \infty$.

Therefore, from (9) and by the properties of the Q function, we have that $P_F' \to q$ as $N \to \infty$. In other words, for any fixed $\epsilon > 0$, $\exists N_3(\epsilon_1)$ such that $|P_F' - q| < \epsilon$ for $N > N_3(\epsilon_1)$. Note that $N_3$ is a function of $\lambda$ and hence a function of $\epsilon_1$.

Define

$$P_{D,1} := (1-q)Q \left( \frac{\lambda - N(\sigma^2_w + \sigma^2_p)}{\sqrt{2N(\sigma^2_w + \sigma^2_p)}} \right),$$

$$P_{D,2} := qQ \left( \frac{(\lambda - M\sigma^2_s - N(\sigma^2_w + \sigma^2_p))/\sqrt{2}}{\sqrt{M\sigma^2_s + 2M\sigma^2_s(\sigma^2_w + \sigma^2_p) + N(\sigma^2_w + \sigma^2_p)^2}} \right).$$

Then we have from (10) that $P'_D = P_{D,1} + P_{D,2}$. It follows from (24) that given any $P'_D, 1 \in (0,1 - q)$, we can solve for $N = 2 \left( \frac{1}{\epsilon_1}Q^{-1}(P'_{D,1}/(1 - q)) \right)^2 = N_3(\epsilon_1).$

Recall that under the weak SU signal assumption we have $M\sigma^2_s > N((1 + \delta)\sigma^2_p + \delta\sigma^2_w)$. Therefore, $\lambda - M\sigma^2_s - N(\sigma^2_w + \sigma^2_p) < N(\epsilon_1 - \delta)(\sigma^2_w + \sigma^2_p) < 0$ since $\epsilon_1 < \delta$, and hence $P_{D,2} \to q$ as $N \to \infty$. In other words, for any $\epsilon_2 > 0$, $\exists N_2$
such that \(|P_{D,2}^r - q| < \epsilon_2\) or more precisely \(q - \epsilon_2 < P_{D,2}^r < q\) for \(N > N_2\) where \(N_2\) is not a function of \(\epsilon_1\). The constraints on \(N\) and \(\epsilon_1\) are illustrated by the shaded area in Fig. 4. Note that the shape of the curve \(N_3(\epsilon_1)\) could be different from the one in Fig. 4 which is only for the purpose of illustration. What we need for the proof is a curve with finite values in the interval \((-\sigma_p^2/(\sigma_p^2 + \sigma_w^2), \delta))\).

Since \(N_2(\epsilon_1) \to \infty\) as \(\epsilon_1 \to 0\), a portion of the \(N_2(\epsilon_1)\) curve will always fall in the shaded area. That is, \(\exists N, \epsilon_1\) such that for any \(P_D^r = P_{D,1}^r + P_{D,2}^r \in (q - \epsilon_2, 1)\) and hence \(P_D^r \in (q, 1)\). Also, since such \(N, \epsilon_1\) are in the shaded area, constraint \(N > N_3(\epsilon_1)\) is met, and thus \(|P_{F}^r - q| < \epsilon\).

**Note:** from the above proof we see that there are an infinite number of values for \(N\) to make Theorem 3.1 hold. These values for \(N\) may not be contiguous due to the constraint (26), but approach infinity.

Following a similar argument, we see that under the conditions of Theorem 3.1, for any \(P_D \in (0, 1)\), \(P_F \to 0\) as \(N \to \infty\). Therefore, the upward line segment of the \(P_D^r - P_F^r\) curve described in Theorem 3.1 lies strictly to the right of the \(P_D^r - P_F^r\) curve, indicating that the \(P_D^r - P_F^r\) curve is inferior to the \(P_D^r - P_F^r\) curve (since for the same probability of detection, the former gives a larger value for the probability of false alarm). The relationship between the two curves is shown in Fig. 5.

Now consider the proposed algorithm, i.e., the \(P_D - P_F\) curve. A properly configured matched filter needs to have \(P_F^m \to 0\) and \(P_D^m \to 1\), which, according to (14) and (15), is achievable if \(\eta = \alpha L\) with constant \(\alpha < c_s^2\sigma_s^2\) and \(L \to \infty\). Otherwise, there will be too many false alarms, which keep the receiver decoding wrong signals and result in the missing of future incoming packets, or the receiver will simply lose most of incoming packets. Since \(P(SR\text{ decides } A_0|h_2) = P(SR\text{ decides } A_0|h_1) = 1 - P_D^m\), we have \(P(SR\text{ decides } A_0|h_2) \to 0\). We also have \(P(SS\text{ decides } H_1|h_0) = P(SS\text{ decides } H_1|h_0) \to 0\), as \(N\) defined in (26) approaches \(\infty\). Therefore, by (22) and under the conditions in Theorem 3.1, we have \(P_F \to 0\).

Also, for a properly configured matched filter and under the assumption that \(c_p^2\sigma_p^2/\sigma_w^2 < 1\), we have \(P(SR\text{ decides } A_0|h_1) = 1\) and \(P(SR\text{ decides } A_0|h_3) \to 0\). Therefore, from (23) we have \(P_D \to 1 - q\). \(P_{D}^r\).

Therefore, the ROC curve of the proposed algorithm overlaps that of the ideal energy detector for a wide range of values for the probability of detection. The relationship between the two curves is shown in Fig. 6. The region of practical importance is the region where the probability of false alarm is negligible. Therefore, for practical purposes, the proposed algorithm achieves the performance of an ideal energy detector for a wide range of values for the probability of detection. For better understanding the performance of the three algorithms, we plot their ROC curves in Fig. 7.

As another example, we consider a scenario where the PU SNR \(\gamma_p\) is as low as -20dB. The other parameters are: \(\gamma_s = 10dB\), \(N = 200,000\), \(M = 1000\), \(L = 100\), \(c_p = 1\), and \(q = 0.02\). The corresponding ROC curves are shown in Fig. 8. Again, similar observations can be made on the merits of the proposed algorithm compared to the energy detection algorithm that ignore the SU transmissions (i.e., the energy detector in the non-ideal case).

**Fig. 4.** Illustration of the existence of \(N\) and \(\epsilon_1\) (and hence \(\lambda\)) that satisfy Theorem 3.1.

**Fig. 5.** The \(P_D^m - P_F^m\) curve (green dotted line with dots) and the \(P_D^r - P_F^r\) curve (blue solid line with crosses), with \(\gamma_p = -10dB\), \(\gamma_s = 10dB\), \(N = 3000\), \(M = 1000\), \(L = 100\), \(c_p = 1\), \(q = 0.05\), and \(P_F^m = 0.01\). The arrows connect two points with thresholds \(\lambda = \zeta\).

**IV. CONCLUSION**

In this paper, we propose to leverage the matched filter that naturally exists in a secondary user (SU) radio devices to improve the performance of energy-detection based spectrum sensing with imperfect coordination in the silent periods. In particular, we consider the challenging case where the SU signal is weak. The analysis shows that, for practical purposes, the proposed algorithm achieves performance close to that of an ideal energy detector without the existence of mistaken SU transmissions. The proposed algorithm can be applied to a vast
range of cognitive radios with minimal additional hardware implementation effort.

APPENDIX

Lemma 1: For the following hypotheses

\[ H_0 : z(n) = w(n), \quad n = 1, \ldots, N \]  
\[ H_1 : z(n) = x(n) + w(n), \quad n = 1, \ldots, N \]

where \( w(n) \) are the i.i.d noise with mean zero and variance \( \sigma_w^2 \) and \( x(n) \), \( n = 1, \ldots, M \) are the i.i.d. signal with mean zero and variance \( \sigma_x^2 := \sum_{n=1}^{M} x(n)^2 / M \) and \( x(n) = 0, n > M \), and

the energy detector \( r := \sum_{n=1}^{N} |z(n)|^2 \overset{H_1}{\geq} \lambda \), the probability of false alarm and the probability of detections are

\[ P_F = Q \left( \frac{\lambda - N\sigma_w^2}{\sqrt{2N\sigma_w^2}} \right) \]
\[ P_D = Q \left( \frac{\lambda - M\sigma_x^2 - N\sigma_w^2}{\sqrt{2(M\sigma_x^2 + 2M\sigma_x^2\sigma_w^2 + N\sigma_w^4)}} \right) \]

where \( \gamma := \sigma_x^2 / \sigma_w^2 \), as \( N \to \infty \).

Proof: We find the distributions of \( r \) under two hypotheses, i.e., \( p(r|H_1) \) and \( p(r|H_0) \). The optimal (in the sense of maximizing the probability of detection for a given probability of false alarm) energy detector is

\[ \Lambda(r) := \frac{p(r|H_1)}{p(r|H_0)} \overset{H_1}{\geq} \eta, \quad (29) \]

where \( \eta \) is a threshold.

We have that under \( H_1 \)

\[ E[r] = N\sigma_w^2 + M\sigma_x^2 \quad (30) \]
\[ E[r^2] = M(M+2)\sigma_x^4 + 2M(N+2)\sigma_x^2\sigma_w^2 + N(N+2)\sigma_w^4 \quad (31) \]
\[ \sigma_r^2 = E[r^2] - E[r]^2 = 2(M\sigma_x^4 + 2M\sigma_x^2\sigma_w^2 + N\sigma_w^4) \quad (32) \]

where in (31) we have used the result that for a random variable \( x \sim \mathcal{N}(0, \sigma^2) \), \( E[x^4] = 3\sigma^4 \), which can be obtained by differentiating \( \alpha \) twice in \( \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha} \) and letting \( \alpha = 1/(2\sigma^2) \).
Similarly, we have that under $H_0$

\[
E[r] = N\sigma_w^2 \quad (33)
\]
\[
E[r^2] = N(N + 2)\sigma_w^4 \quad (34)
\]
\[
\sigma_r^2 = E[r^2] - E[r]^2 = 2N\sigma_w^4. \quad (35)
\]

It is easily seen that under $H_0$, $r$ approaches Gaussian and $P(r|H_0) \sim N(N\sigma_w^2, 2N\sigma_w^4)$. Under $H_1$, $\sum_{n=1}^N(x(n) + w(n))^2$ and $\sum_{n=M+1}^N(x(n) + w(n))^2 = \sum_{n=M+1}^N w(n)^2$ are independent, and by the Central Limit Theorem, both approach Gaussian for large $M$ and large $N - M$, and thus $r$ approaches Gaussian and $P(r|H_1) \sim N(M\sigma_x^2 + N\sigma_w^2, 2(M\sigma_x^4 + 2M\sigma_x^2\sigma_w^2 + N\sigma_w^4))$. It can be checked that (29) reduces to

\[
\begin{align*}
&H_1 \quad r \geq \lambda, \\
&H_0 \quad P_F = Q\left(\frac{\gamma - L\sigma_x^2}{\sqrt{L(\sigma_w^2 + \sigma_y^2)}}\right), \\
&H_0 \quad P_D = Q\left(\frac{\gamma - L\sigma_x^2}{\sqrt{L(\sigma_w^2 + \sigma_y^2)}}\right),
\end{align*}
\]

where $\gamma = \sigma_x^2/(\sigma_w^2 + \sigma_y^2)$, as $L \to \infty$.

**Proof:** As $L \to \infty$, by the Central Limit Theorem, $T(z)$ is Gaussian distributed under either hypothesis. We have that under $H_1$

\[
E[T(z)] = \frac{\sigma_x^2}{\sigma_w^2}\sigma_x^2 \quad (40)
\]
\[
\sigma_r^2 = L(\sigma_w^2 + \sigma_y^2)\sigma_x^2 \quad (41)
\]

and that under $H_0$

\[
E[T(z)] = 0 \quad (42)
\]
\[
\sigma_r^2 = L\sigma_w^2\sigma_x^2 \quad (43)
\]

Therefore,

\[
\begin{align*}
P_F &= \int_{\gamma}^{\infty} P(T|H_0) dT = \int_{\gamma}^{\infty} \exp\left(-\frac{(T-L\sigma_x^2)^2}{2L(\sigma_w^2 + \sigma_y^2)\sigma_x^2}\right) dT \\
&= Q\left(\frac{\gamma - L\sigma_x^2}{\sqrt{L(\sigma_w^2 + \sigma_y^2)}}\right),
\end{align*}
\]

where $\gamma = \sigma_x^2/(\sigma_w^2 + \sigma_y^2)$.

Note that if $M = N$, we have $P_D = Q((Q^{-1}(P_F) - \sqrt{N/2})/(1 + \gamma))$, which agrees with the result in [5].

Note also that the assumption on the statistics of the noise is very general. For example, we do not assume that the noise is Gaussian. Therefore, the noise can include an interference signal, which we use in the derivations for the energy detector in the non-ideal case.

**Lemma 2:** For the following hypotheses

\[
H_0: z(n) = w(n), \quad n = 1, ..., L \quad (38)
\]
\[
H_1: z(n) = x(n) + y(n) + w(n), \quad n = 1, ..., L \quad (39)
\]

where $w(n)$ are i.i.d Gaussian $\mathcal{N}(0, \sigma_w^2)$, $y(n)$ are i.i.d with mean zero and variance $\sigma_y^2$, and $x(n)$ are i.i.d with variance $\sigma_x^2$ and known to the receiver, and $w(n), y(n)$ and $x(n)$ are independent, and the matched filter $T(z) := \sum_{i=1}^L z(i)x(i)$ $\geq \gamma$, $H_0$ the probability of false alarm and the probability of detection are

\[
\begin{align*}
P_F &= \int_{Q^{-1}(\gamma)}^{\infty} P(T|H_0) dT = \int_{Q^{-1}(\gamma)}^{\infty} \exp\left(-\frac{(T-L\sigma_x^2)^2}{2L(\sigma_w^2 + \sigma_y^2)\sigma_x^2}\right) dT \\
&= Q\left(\frac{\gamma - L\sigma_x^2}{\sqrt{L(\sigma_w^2 + \sigma_y^2)}}\right),
\end{align*}
\]

where $\gamma = \sigma_x^2/(\sigma_w^2 + \sigma_y^2)$.

Note that Lemma 2 is different from the matched filter in [6, p95] in that Lemma 2 also considers an interference signal $y(n)$ which is not necessarily Gaussian. When $y(n) = 0$, Lemma 2 agrees with the result in [6, p95].

**REFERENCES**


